Grades 5-6

1. Alex wrote nine different positive digits on one red card and eight blue cards, only one digit per card. The product of all the digits on the blue cards is 72576.

What digit is written on the red card?

2. Marcus painted the six faces of a $7 \times 7 \times 7$ wood cube red, and then sawed the cube into $1 \times 1 \times 1$ cubes. After that, he took the cubes with at least two red faces and stacked them in a tower with a 2×2 base.

Find the height of this tower.

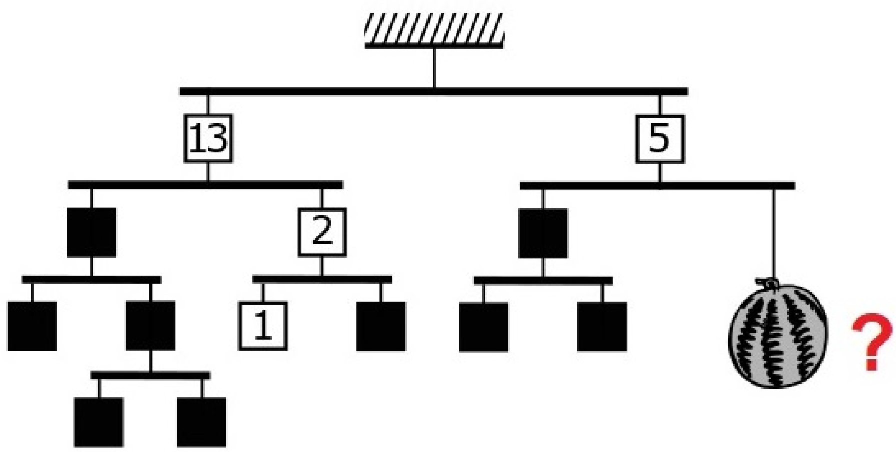
3. Find the sum of all the digits of the result of the calculation:

$$2^2 \times 4^4 \times 5^5 - 1$$

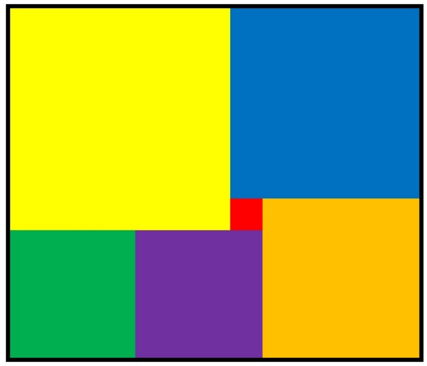
4. The hanger shown in the diagram is balanced. The weights of the white boxes are marked in the figure. The weights of the black boxes are unknown; they could be the same or they could be different.

Find the weight of the watermelon.

Assume that all parts of the hanger itself (horizontal bars and vertical strings) weigh nothing.



5. The paper rectangle is divided into six non-overlapping squares as shown in the diagram. Compute the area of the rectangle if the area of the smallest square equals 1.



6. There are 100 puppies at a "Learn with Your Pet" event. Some are pure white, some are pure black, and the rest are *tri*-colored with one white, one black, and two brown paws each. The puppies have 221 white and 145 black paws altogether.

How many *tri*-colored puppies are at the event?

7. Nick has \$5, \$20, and \$100 bills that total 17 bills. He exchanges all his \$5 bills for \$1 bills, then all his \$20 bills for \$5 bills, and finally all his \$100 bills for \$20 bills. At the end, Nick has a total of 77 bills.

How many \$20 bills did Nick have originally?

Assume that each exchange preserves the total dollar amount of Nick's money.

8. The product of a two-digit number and a three-digit number is equal to a five-digit number as shown in the multiplication example.

Find the five-digit number (the product) if the tens digit of the three-digit number equals 8.

(Leftmost stars in each of the six numbers shown in the example can represent any non-zero digits, other stars can represent any digits.)

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        * *
×     * 8 *
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      * * *
      * *
  * * *
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  * * * * *
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9. Lucie and Suzie buy greeting cards. Lucie buys 10 more cards than one third of the girls' total.
How many more cards should Suzie buy so that she has 10 more cards than two thirds of the girls' total?
Assume that Lucie does not make any purchases after her initial purchase.

10. Each of six jars contains the same number of candies. Alice moves half of the candies from the first jar to the second jar. Then Boris moves half of the candies from the second jar to the third jar. Then Clara moves half of the candies from the third jar to the fourth jar. Then Dara moves half of the candies from the fourth jar to the fifth. Finally, Ed moves half of the candies from the fifth jar to the sixth jar. At the end, 30 candies are in the fourth jar.
How many candies are now in the sixth jar?

11. In the "expression"

$$1 @ 2 @ 3 @ 4 @ 5 @ 6 @ 7 @ 8 @ 9 @ 10 @ 11 @ 12$$

each of the eleven **@** symbols is replaced with either × or ÷ such that the value of the resulting expression is an integer.
Find the greatest common factor of all such integer values.

12. The diagram shows three famous RSM cafes, ***R***, ***S***, and ***M***. It takes one step to move from one of these cafes directly to another.
How many different ways are there to start at ***R*** and end at ***M*** in exactly seven steps?
One possible way is

$$\boldsymbol{R - M - R - S - M - R - S - M}.$$