MPOWER!
FOR KIDS WHO LIKE TO **SOLVE PROBLEMS**
ISSUE 2 | DEC 2015

ANSWERS

THE THREE PRINCESSES (PAGE 1)

Answer:
He asks any one of the princesses (call her Princess A): Is Princess B older than Princess C? If the princess answers yes, then he chooses Princess C; if she answers no, then he chooses Princess B. The prince does not choose the princess he asks.

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FACTORS & FAULTY LIGHTS (PAGE 2-4)

Answer:

Problem 1

The lights will blink for 1000 seconds, or minutes; so the blinking would have stopped at 12:16:40 A.M., or 12:17 A.M. (to the nearest minute). Since there are 31 square numbers from 1 to 1000 (312 = 961, 322 = 1024), 31 bulbs would have been on when the blinking stopped.

Problem 2

There are 15 square numbers from 12 = 1 to 152 = 225, which means that there are at least 225 bulbs on the string of lights. However, for each number of bulbs between 152 and 162 = 256 (the next square number), we get the same result. So, the number of bulbs on the string of lights is any number from 225 to 255 (inclusive).

Problem 3

There are 9 powers of 2 in range from 1 to 300: 1=20, 2=21,..., 256=28. After Step 1 (1st second) all the bulbs will be on. After Step 2 (2nd second), all even-numbered bulbs turn off. Thus, there are 300 – 150 = 150 odd bulbs that remain on. In Step 3 (4th second), all the bulbs with numbers that are divisible by 4=22 will turn on again. There are 75 such bulbs. The remaining 75 bulbs from Step 2 will not turn on again since these bulbs are divisible by 2=21 only. In Step 4 (8th second), all the bulbs with numbers divisible by 8=23

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AMUSEMENTS (PAGES 5 & 7)

Answers:

1) Saturday

2) A = 2; B = 1; C = 7; D = 8

3) B : C = 9 : 4

4) 204

5) 6 days

6) 256 students

7) 12

ANSWERS

THE SECRET HIDEOUT (PAGES 8 & 9)

Answer:

The password Oliver should have given is five.

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M CHALLENGE (PAGES 10 & 11)

Answers:

1) Ezra has 32 more pencils than Joe.

2) 20 blocks

3) 888

4) 3

5) 36 square centimeters

6) 499

7) $A = \frac{2b^2}{a}$

8) 8) $x = -2$, $x = -1$, $x = 1$, $x = 3$

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Oops! Did you spot the error in the hard copy of MPower!?
Well, we did. The correct definition of the Geometric Mean is a/G = G/b.

PYTHAGOREAN MEANS (PAGES 12 & 13)

Answer:

Note: We consider the means for only positive values of real numbers a and b.

Problem 1

H = G = A when a = b. (AO = OB on the figure.)

Problem 2

(a) The arithmetic mean of 12 and 6 is $A=\frac{12+6}{2}=\frac{18}{2}=9.$

(b) The harmonic mean of 12 and 6 is $H=\frac{2\cdot 12\cdot 6}{12+6}=\frac{18\cdot 8}{18}=8.$

Problem 3

Any equilateral triangle will have one side that is the arithmetic, geometric, and harmonic mean of the other two sides. For example, the equilateral triangle with sides of length 3, 3, and 3:

$$A=\frac{3+3}{2}=\frac{6}{2}=3;\quad G=\sqrt{3\cdot 3}=\sqrt{3^2}=3;\quad H=\frac{2\cdot 3\cdot 3}{3+3}=\frac{18}{6}=3$$

Problem 4

(1) Proof that $\frac{a+b}{2}\geq\sqrt{ab}$.

$$\frac{a+b}{2}-\sqrt{ab}=\frac{a+b-2\sqrt{ab}}{2}\geq 0$$

Using the formula of the square of the difference of two numbers, $(x+y)^2$, and letting $x=(\sqrt{a})^2$ and $y=(\sqrt{b})^2$ gives:

$$\frac{a+b-2\sqrt{ab}}{2}=\frac{(\sqrt{a})^2+(\sqrt{b})^2-2\sqrt{a}\cdot\sqrt{b}}{2}=\frac{(\sqrt{a}-\sqrt{b})^2}{2}\geq 0 \qquad \text{(True)}$$

So, $\frac{a+b}{2}-\sqrt{ab}\geq 0$; and, thus, $\frac{a+b}{2}\geq\sqrt{ab}$.

(2) Proof that $\sqrt{ab}\geq\frac{2ab}{a+b}$.

$$\sqrt{ab}\div\frac{2ab}{a+b}=\sqrt{ab}\cdot\frac{a+b}{2ab}=\frac{a+b}{2\sqrt{ab}}=\frac{a+b}{2}\div\sqrt{ab}\geq 1$$

From (1) $\frac{a+b}{2}\div\sqrt{ab}\geq 1$ is true. So $\sqrt{ab}\div\frac{2ab}{a+b}\geq 1$ is true; and, thus, $\sqrt{ab}\geq\frac{2ab}{a+b}$.